ADVANCES IN UPDATING INPUT-OUTPUT TABLES: ITS RELEVANCE FOR THE ANALYSIS OF REGIONAL ECONOMIES*

AVANÇOS NA ATUALIZAÇÃO DE QUADROS *INPUT-OUTPUT*: SUA RELEVÂNCIA PARA A ANÁLISE DE ECONOMIAS REGIONAIS

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ABSTRACT/RESUMO

It is very usual to perform economic impact analysis using the latest input-output tables available. However, the problem arises when, as time goes by, reference data may be somewhat outdated. Nevertheless traditional economic analysis considers it almost a fixed dataset; hence there appears a gap between true and estimated results that should be minimized as far as possible. Therefore, updating input-output matrices is necessary and it helps us achieve better results in terms of reliability. This paper presents a technique of global adjustment that can be applied in scenarios with partial information. It is an algorithm that can be implemented almost in real time, representing a remarkable improvement by avoiding the common delays in this field. In addition, for its performance it is only necessary to know (or predict) the behavior of certain vector quantities. For practical purposes, we present some data of the updated domestic symmetric table of Galicia for 2007, based on the symmetric table for 2005 published by the Galician Statistical Institute (IGE). We also highlight the convergent nature of the proposed procedure.

É muito comum fazer análises aos impactos económicos utilizando as mais recentes matrizes *input-output* disponíveis. Contudo, o problema surge quando, com o decurso do tempo, os dados de referência ficam desatualizados. Não obstante, a análise económica tradicional considera-os uma base de dados quase fixa, levando a que surja uma lacuna entre os resultados estimados e os resultados reais, lacuna essa que deve ser minimizada tanto quanto possível. Portanto, a atualização das matrizes *input-output* é necessária, ajudando a alcançar melhores resultados em termos de fiabilidade dos dados. Este artigo apresenta uma técnica de ajustamento global que pode ser utilizada em cenários onde há informação parcial. Trata-se de um algoritmo que pode ser implementado quase em tempo real, representando uma melhoria considerável ao evitar atrasos comuns neste campo. Além disso, para o seu desempenho só é necessário saber (ou prever) o comportamento de determinadas magnitudes vetoriais. Para efeitos práticos, apresentamos alguns dados referentes à atualização do quadro simétrico (de fluxos domésticos) da Galiza para 2007, tendo

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como base a tabela obtida de forma direta para o ano de 2005. Também se realça o caráter convergente do procedimento proposto.

Keywords: Economic Impact, Galicia, Input-Output Models, Matrix Adjustment, Partial Information.

JEL Codes: C65, C89, D57

Palavras-chave: Impacto Económico, Galiza, Modelos *Input-Output*, Ajustamento da Matriz, Informação Parcial.

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1. INTRODUCTION: UPDATING INPUT-OUTPUT TABLES

The elaboration of input-output tables (IOT) implies a high cost for any national statistical institute, a circumstance that is intensified for regions due to lower budgets. In fact, IOT are often published every five years since it is not usual to have annual versions obtained in a *survey* manner. Despite this lack of information, this productive structure is used to perform different macroeconomic quantifications, among others, the GDP. This means it is likely the achieved estimation will deviate from reality, even in short periods with few structural changes. This is the reason why, as far as possible, the updating techniques are applied. Therefore, an attempt is made to get *non-survey* IOT, or at least the technical coefficient matrices, in order to reduce the deviations we stated above.

The necessity of finding some indirect estimation method that provides a reliable alternative to survey techniques makes the study of updating IOT a topic of great interest. Among the proposed procedures, we can highlight the well-known RAS method as one of the most employed for these adjustments. The RAS is a biproportional method of adjustment, which consists of repeatedly multiplying the elements of the rows and columns in a matrix by correction coefficients. This procedure was proposed by Stone and Brown (1962), but over time, its references and extensions have increased considerably, for example, among others: Bacharach (1970), Allen and Lecomber (1975) and Szyrmer (1989). Although this method has been widely used, there are other alternatives which were presented and compared in several papers, such as: Pavia *et al.* (2009), Lahr and Mesnard (2004) or Jackson and Murray (2004).

Probably the most restrictive limitation of the simple or basic RAS is the forced requirement of knowing the sums of the rows and columns of the matrices subject to the adjustment process in advance. There is a need to break with this traditional approach, going further than the border of the intermediate consumption matrix (the sums of rows and columns of its elements). It is true that the intersectoral relations are very important in themselves; however, they are connected to a system: the whole input-output framework. Thus, any variable belonging to this set can be useful in the adjustment procedure.

The main objective of this paper is to design a technique in which the final result, i.e. the updated OIT, can verify the accounting relationships and, at the same time, satisfy the conditional constraints imposed by the availability of information. In short, the aim is to show that these (partial) updating methods that we propose are suitable techniques if we consider them from broader points of view.

For practical purposes, we are going to update the (domestic) intermediate consumption matrix of Galicia using the proposed method. In this region, as in many others (regions or countries) with aggregate economic accounts that follow the recommendations of Eurostat, data is published data about production, value added and intermediate consumption by industry with a very small time lag. Nevertheless, it is not possible to have sectorally disaggregated data for the different demand divisions (final and intermediate), except for the years that IOT is estimated through direct methods. Therefore, this ignorance of the intermediate demand vector justifies the implementation of the suggested technique.

2. A GLOBAL PROCEDURE TO UPDATE INPUT-OUTPUT TABLES

2.1. DATA REQUIREMENTS FOR THE IMPLEMENTATION OF THE METHODOLOGY

Before starting with the description of the updating technique it is necessary to remember the vectors and matrices that make up the symmetric table (ST)[1]. For the application of the procedure it is necessary to know the ST of the base year (0), its elements are symbolized as follows[2]:

$X(0)$ – intermediate consumption matrix $(n\times n)$.
$Y(0)$– final demand matrix $(n\times f)$.
$u(0)$ – intermediate inputs vector $(n\times 1)$.
$w(0)$ – intermediate demand vector $(n\times 1)$.
$v(0)$ – value added vector by industries $(n\times 1)$.
$m(0)$ – imports vector $(n\times 1)$.

[1] To simplify the presentation of the proposed method, we are not going to make any distinction between domestic and imported flows, although it could be done.

[2] Hereinafter, n is the number of industries or products, and f is the number of components of the final demand vector. Furthermore, i is a column matrix of ones, the superscript ^{T} means matrix transposition and $^{-1}$ the inverse. The notation ^ refers to the diagonalization of a vector.

x(0) – production vector *(n×1)*.

At the same time, one needs to know some data of the year (1) for which the TS will be updated. Specifically, these data need to be known:

x(1) – production vector *(n×1)*.

v(1) – value added vector *(n×1)*. As a difference between *x(1)* and *v(1)*, *u(1)* is obtained.

$z(1) = Y(1)^T i$ – totals of the components of the final demand vector *(f×1)*. Thus, the growth rates vector of the totals of the components of the final demand[3] is known:

$$g^z = \left([\hat{z}(0)]^{-1}\hat{z}(1)\right)i. \tag{1}$$

i´m(1) – total imports *(1×1)*. In other words, the growth rate of the imports is known:

$$g^m = \frac{i^T m(1)}{i^T m(0)}. \tag{2}$$

The update may affect either the IOT as a whole or only a part of it. In fact, if the purpose is to update just the intermediate consumption matrix, the performance of this proposed method should be superior, since too much information is not necessary.

In relation to the required data for the updated year (1), the growth rate of imports (or alternatively the growth rate of the total aggregate final demand) can be ignored because it can be deduced from other specified rates.

The totals of rows and columns of any matrix should be the same. In this way, for the intermediate consumption matrix it is known that:

$$\sum_{i=1}^{n} w_i = \sum_{i=1}^{n} u_i . \tag{3}$$

Briefly, the basic accounting (macroeconomic) relations are the following:

$$x + m = w + y, \tag{4}$$

the supply (production and imports) are equal to the employs (intermediate demand and final demand).

$$x = u + v, \tag{5}$$

and the production should be equal to the sum of the inputs.

If the components of the vectors in (4) are added then:

$$\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} m_i = \sum_{i=1}^{n} w_i + \sum_{i=1}^{n} y_i , \tag{6}$$

But taking into account (3) we obtain

$$\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} m_i = \sum_{i=1}^{n} u_i + \sum_{i=1}^{n} y_i . \tag{7}$$

From (5) we get

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} u_i + \sum_{i=1}^{n} v_i . \tag{8}$$

[3] We keep the notation *g* to symbolize the different growth rates, the superscripts related to *g* will refer to the corresponding vector variable.

Then you can directly substitute in (7)

$$\sum_{i=1}^{n} u_i + \sum_{i=1}^{n} v_i + \sum_{i=1}^{n} m_i = \sum_{i=1}^{n} u_i + \sum_{i=1}^{n} y_i , \tag{9}$$

once the obvious simplification it is done and knowing that $\sum_{i=1}^{n} y_i = \sum_{j=1}^{f} z_j$ then:

$$\sum_{i=1}^{n} v_i + \sum_{i=1}^{n} m_i = \sum_{j=1}^{f} z_j \tag{10}$$

So,

$$i^T m = \sum_{i=1}^{n} m_i = \sum_{j=1}^{f} z_j - \sum_{i=1}^{n} v_i . \tag{11}$$

In some cases it is better to clear the total (aggregate) final demand. In fact, this will be our procedure in the application part of this paper.

For a certain year, for example the base year, it is evident that the total production, domestic and imported, is equal to the sum of intermediate demand and final demand:

$$x(0) + m(0) = X(0)i + Y(0)i = w(0) + y(0). \tag{12}$$

From the other point of view, production by industries is equal to the sum of intermediate and primary inputs:

$$x(0) = [X(0)]^T i + v(0) = u(0) + v(0). \tag{13}$$

Following the definition of technical coefficients, a_{ij}, it is known that $A = X\hat{x}^{-1}$.

2.2. THE POSSIBILITY OF UPDATING A ST WITH PARTIAL INFORMATION

If the production, the value added and the totals of the components of the final demand vector growth rates are known, is possible to update ST. Eurostat (2008) explains how to proceed in a similar scenario, although for supply-use tables, through what was called the Euro method (EU), developed by Beutel (2002). The EU responds to a global approach but can only be applied to squared matrices and sometimes it is not convergent, see Temurshoev *et al.* (2011).

Here is where we suggest an adjustment technique based on the common accounting relations of the IO analysis and which, at the same time, rests on the RAS, or another partial adjustment technique. The adjustments are made continuously and in parallel on the intermediate consumption and the final demand matrices with the only constraint being the availability of information. Of course, the simple RAS could not be applied with the data that we stated above, because the sum by rows of the intermediate consumption matrix is unknown; Temurshoev *et al.* (2011).

The global procedure is based on a simple idea which involves making estimations using two paths specified by two working hypotheses: the stability of technical coefficients and the balance between supply and demand in each industry. In this method the intermediate demand vector for these two paths is repeatedly estimated and the differences between these two estimations checked. These differences that appear in each phase are distributed in both directions depending on the weight of the intermediate consumption by row.

Systematically, in this formulation, correction coefficients will be used. At first, the elements of the main diagonal of the correction coefficient matrices match with some of the growth rates specified before. After that, as the iteration by rows progresses, these elements are going to be elaborated according the last two intermediate demand and final demand estimations. This means that the row constraints are variables and the column constraints are fixed; this is the reason why the dynamics of this adjustment perform differently in one case then in the other. Moreover, this condition highlighted here represents a novelty and a methodological change in this area of research.

Regarding the approach paths, hereinafter symbolized with R and S; these will follow opposite directions. In general, within the updating IOT context it is assumed that changes in variables or in the elements of the matrices are minimal. But still, if any growth rate is too far from the unity the obtained results may be at least questionable and should be interpreted with caution.

The process finishes when there is an acceptable approximation, i.e. when the differences between the two estimations of the intermediate demand are zero or practically zero. Thus, in every moment the intermediate demand vector is taken as a test variable. It is expected that as the number of iterations increases the convergence of the elements of the IOT appears simultaneously, and it is so. As a consequence, it will be shown how the convergence of the different estimations is an important property of this technique.

2.3. A TECHNIQUE THAT DISTRIBUTES THE DIFFERENCES AS A "REBOUND EFFECT"

In the first place, the intermediate consumption matrix is estimated assuming the stability of the columns of the ST, i.e. to modify the initial matrix, $X(0)$, taking into account, the growth rates of the intermediate inputs, g^u.

$$X^{(1)} = X(0)\hat{g}^u. \tag{14}$$

In other words, what we are doing here is to implement one of the phases of the RAS. This will also be the procedure in the next corrections by columns. So, at this point it is possible to obtain the first estimation of the intermediate demand vector through the S path: $w^{S(1)} = X^{(1)}i$.

From the production vector, $x(1)$, the other estimation of the intermediate demand, $w^{R(1)}$, could be obtained. This means that is possible to estimate the final demand matrix, $Y^{(1)}$, through a double adjustment by rows and columns. It is necessary to guarantee the sum by columns, which is indicated by the growth rate of the total of the components of the final demand, g^z. It is expressed as follows:

$$y^{R(1)} = Y^{(1)}i = [(\hat{g}^x Y(0))\hat{g}^z]i. \tag{15}$$

Although this first proportional correction is made based on the change of the production, as the process progresses there appear other kind of rectifications that will smoothen this first effect.

Therefore, it is also possible to get an estimation of the intermediate demand from the R path: $w^{R(1)} = x^{(1)}i - y^{R(1)}$. However, in general $w^{S(1)}$ and $w^{R(1)}$ do not match. Some of the components are overestimated and the others are underestimated using one of the paths and vice versa. According to this, we can define the following vector of differences:

$d_w^{(1)} = w^{R(1)} - w^{S(1)}$, where $\sum_{i=1}^{n} d_{wi}^{(1)} = 0$ taking into account that

$$\sum_{i=1}^{n} w_i^{R(1)} = \sum_{i=1}^{n} w_i^{S(1)}. \tag{16}$$

The elements of this vector can be positive or negative but their sum must be equal to zero, i.e. the overestimations are compensated with the underestimations.

Therefore, there is a gap, which is motivated by the two different paths of estimation, R and S. So, it is appropriate to correct the intermediate consumption and the final demand matrices sharing in both ways the differences obtained[4]. As a "rebound effect", the intermediate consumption matrix that we estimated in the last step is modified, $X^{(1)}$, depending on the weight of these consumptions over the total by rows:

$$X^{(2)} = X^{(1)} + C\hat{d}_w^{(1)} \left(\hat{w}^{S(1)}\right)^{-1} X^{(1)}, \tag{17}$$

The coefficient selected for each row is constant,

$$c_i = C, \quad i = 1,2,\ldots,n.$$

Hence

$$\sum_{i=1}^{n} Cd_{wi}^{(1)} = C\sum_{i=1}^{n} d_{wi}^{(1)} = 0. \tag{18}$$

In general, the scalars that multiply these differences will lead to different estimations of the intermediate demand vectors (depending on the assigned values), but these vectors must always satisfy the relation (3).

Although it is a strong simplification, it is also possible to multiply each component of the vector of differences by a different scalar, always taking into account the following constraint:

$$\sum_{i=1}^{n} c_i d_{wi}^{(1)} = 0, \tag{19}$$

as the balance of the system cannot be altered.

So, the scalar $c_i d_{wi}^{(1)}$ is the parameter that influences the elements of the row i in this phase. Regarding this aspect, we found the explanation of Callealta and López (2005) on

[4] The selection of the sharing criteria is very important and it is not unique, obviously.

the proposed modification of the RAS made by Bachem and Korte (1979) very illustrative.

With additional information about the trends of the uses by product (or industry) the simplification that we indicated before could be avoided. *C* is a scalar that belongs to the interval [0, 1], and that refers to the sharing criteria. In this case, we chose half of the difference, i.e. *C*=0,5. Of course, we could select another coefficient but this one is more convenient for the description[5]. More complex ways for the distribution of the errors in the adjustment[6] could also be used.

Actually, what is done in this next step is to apply a particular RAS, which is conditioned by the available data. If additional information is available that can ensure higher productive structure stability to the final demand matrix, this parameter would decrease[7].

The characteristic element of $X^{(2)}$ is as follows:

$$x_{ij}^{(2)} = x_{ij}^{(1)} + 0.5x_{ij}^{(1)}\frac{w_i^{R(1)} - w_i^{S(1)}}{w_i^{S(1)}}, \quad i,j = 1,2,\ldots,n. \quad (20)$$

Taking this into account, it is possible to get the following expression:

$$x_{ij}^{(2)} = \frac{x_{ij}^{(1)}w_i^{S(1)} + 0.5x_{ij}^{(1)}w_i^{R(1)} - 0.5x_{ij}^{(1)}w_i^{S(1)}}{w_i^{S(1)}} =$$

$$= 0.5\frac{x_{ij}^{(1)}w_i^{S(1)} + x_{ij}^{(1)}w_i^{R(1)}}{w_i^{S(1)}}. \quad (21)$$

In order to give an easier description and to show the similarities with the RAS, it can also be defined as:

$$x_{ij}^{(2)} = x_{ij}^{(1)}\frac{\overline{w}_i^{(1)}}{w_i^{S(1)}}, \quad i,j = 1,2,\ldots,n. \quad (22)$$

Where $\overline{w}_i^{(1)} = \frac{w_i^{R(1)} + w_i^{S(1)}}{2}$ is the arithmetic average of the estimations of the components of the intermediate demand vector obtained through the two paths described. Since the components of these vectors do not match, we chose to consider an intermediate value, assuming that somehow the error is distributed in a certain way, which is not necessarily the most appropriate.

Advancing in the iterative process, the two estimated vectors start to approach one another: $w_i^{R(n)} \approx w_i^{S(n)}$.

In this way, the matrix $X^{(2)}$, could be alternatively expressed as indicated:

$$X^{(2)} = \frac{1}{2}\left(\hat{w}^{R(1)} + \hat{w}^{S(1)}\right)\left(\hat{w}^{S(1)}\right)^{-1} X^{(1)} =$$

$$= \hat{\overline{w}}^{(1)}\left(\hat{w}^{S(1)}\right)^{-1} X^{(1)}. \quad (23)$$

This last expression (23) helps in the explanation of the idea of the distribution of differences that we stated before. At the same time, it is indicated how the ignorance of the sum by rows of the intermediate consumption matrix is a constraint that can be overcome in the application of the RAS technique, with this proposed method.

At this point, everything indicates that it seems reasonable to use the corrections as stated and not according to growth rates that come from the latest estimations of the intermediate demand. This would mean to consider one of the estimations as correct when you know that it is not.

Making the corrections based on estimations of the production is also undesirable. The production can vary in a different way than the intermediate demand (presenting different growth rates). This is probably the reason why the method proposed by Beutel is not always able to ensure convergence.

Focusing on the intermediate inputs vector and estimations of it that can be obtained, $u(1) = [X^{(1)}]^T i$ and $u^{R(1)} = [X^{(2)}]^T i$ respectively, one sees that they are different. Thus, it is possible to estimate another matrix of intermediate consumptions, $X^{(2c)}$, as follows[8]:

$$X^{(2c)} = X^{(2)}\left(\hat{u}^{R(1)}\right)^{-1}\hat{u}(1). \quad (24)$$

[5] It is important to keep in mind that it is not possible to distribute an amount greater than the available resources; this could imply that for some row one might need to change the sharing criteria, as an exception.

[6] It could be indicated that the first phase of the simple RAS can be presented in an alternative manner. More concretely, the rectification over the intermediate consumption matrix is expressed in matrix format based on $X^{(1)} = RX_0$, where $R = (\hat{w}_t)\left(\hat{w}_0\right)^{-1}$. The generic element of this first estimation can be alternatively expressed as:

$$x_{ij}^{(1)} = x_{ij}(0) + x_{ij}(0)\frac{w_t - w_0}{w_0}, \quad i,j = 1,2,\ldots,n,$$

And once it is simplified we could obtain the more familiar expressions:

$$x_{ij}^{(1)} = x_{ij}(0)\frac{w_t}{w_0}, \quad i,j = 1,2,\ldots,n.$$

[7] It is important to remember the role of prices. Apparent changes in the productive structure could be changes in prices.

[8] It is not necessary, but in this case the difference between the real data and the last estimation can be defined

$d_u^{(1)} = u(1) - u^{R(1)}$, where $\sum_{i=1}^{n} d_{ui}^{(1)} = 0$.

Within this adjustment process the correction of the intermediate consumption matrix by columns appears. So, similar to the correction by rows, we obtain

$$X^{(2c)} = X^{(2)} + X^{(2)}\left(\hat{u}^{R(1)}\right)^{-1}\hat{d}_u^{(1)}.$$

The generic element of $X^{(2c)}$ is as follows:

$$x_{ij}^{(2c)} = x_{ij}^{(2)} + x_{ij}^{(2)}\frac{u_j(1) - u_j^{R(1)}}{u_j^{R(1)}}, \quad i,j = 1,2,\ldots,n.$$

Or alternatively:

$$x_{ij}^{(2c)} = x_{ij}^{(2)}\frac{u_j(1)}{u_j^{R(1)}}, \quad i,j = 1,2,\ldots,n.$$

In other words, the simple RAS is applied by columns. Advancing in the procedure we will get that $u_j(1) \approx u_j^{R(n)}$.

At the same time, we need to estimate the final demand matrix. According to the idea of the double distribution, we obtain an estimation of the final demand vector, $y^{S(1)}$, as a difference between the production, $x^{(1)}$, and the last intermediate demand vector, $\overline{w}^{(1)}$.

Those sectors that do not supply intermediate inputs, due to their nature, should have a special treatment because they cause inconsistencies, i.e. final demand cannot exceed production[9]. Nevertheless, they can be avoided using appropriate aggregations.

Therefore, we obtain the matrix

$$Y^{(2)} = [\hat{y}^{S(1)}(\hat{y}^{R(1)})^{-1}Y^{(1)}]\hat{z}(1)(\hat{z}^{(1)})^{-1}, \qquad (25)$$

where $z^{(1)} = [\hat{y}^{S(1)}(\hat{y}^{R(1)})^{-1}Y^{(1)}]^T i$.

So $w^{R(2)} = x(1) - y^{R(2)}$, and of course $y^{R(2)} = Y^{(2)}i$. The other estimation path gives us $w^{S(2)} = X^{(2c)}i$. These two vectors are closer than before but with these iterative phases it is normal that they do not match. Then, the difference between them appears again:

$$d_w^{(2)} = w^{R(2)} - w^{S(2)}. \qquad (26)$$

And a new estimation of the intermediate consumption matrix:

$$X^{(3)} = X^{(2c)} + 0.5\hat{d}_w^{(2)}\left(\hat{w}^{S(2)}\right)^{-1} X^{(2c)} \qquad (27)$$

or $X^{(3)} = \hat{\overline{w}}^{(2)}\left(\hat{w}^{S(2)}\right)^{-1} X^{(2c)}$, where $\overline{w}^{(2)} = \frac{1}{2}(w^{R(2)} + w^{S(2)})$.

Now by columns, $X^{(3c)} = X^{(3)}\left(\hat{u}^{R(2)}\right)^{-1}\hat{u}(1)$.

Finally, and considering the expressions we stated before, it is possible to highlight the generic matrix formulations. For the final demand:

$$Y^{(n)} = [\hat{y}^{S(n-1)}(\hat{y}^{R(n-1)})^{-1}Y^{(n-1)}]\hat{z}(1)(\hat{z}^{(n-1)})^{-1}, \qquad (28)$$

where $z^{(n-1)} = [\hat{y}^{S(n-1)}(\hat{y}^{R(n-1)})^{-1}Y^{(n-1)}]^T i$.

Then, $w^{R(n)} = x(1) - y^{R(n)}$, knowing that $y^{R(n)} = Y^{(n)}i$. The last estimation of the final demand was $w^{S(n)} = X^{(nc)}i$. And the difference $d_w^{(n)} = w^{R(n)} - w^{S(n)}$ is used to obtain a new estimation of the intermediate consumptions matrix:

$$X^{(n+1)} = X^{(nc)} + 0.5\hat{d}_w^{(n)}\left(\hat{w}^{S(n)}\right)^{-1} X^{(nc)} =$$

$$= \hat{\overline{w}}^{(n)}\left(\hat{w}^{S(n)}\right)^{-1} X^{(nc)}, \qquad (29)$$

where $\overline{w}^{(n)} = \frac{1}{2}(w^{R(n)} + w^{S(n)})$.

Therefore, we come across another backwards rectification in order to correct the columns of the intermediate consumption matrix:

$$X^{((n+1)c)} = X^{(n+1)}\left(\hat{u}^{R(n)}\right)^{-1}\hat{u}(1). \qquad (30)$$

[9] Common examples are: *Public Administration* or *Non-market activities.*

The process finishes when there is an acceptable approximation, i.e. when the differences between the two estimations of the intermediate demand are zero or practically zero. At the same time, the compatibility with the available data is guaranteed.

3. AN EMPIRICAL APPLICATION: UPDATING THE DOMESTIC SYMMETRIC INPUT-OUTPUT TABLE OF GALICIA

In order to emphasize the applied importance of the proposed procedure, we are going to update the intermediate consumption matrix (domestic flows) of the ST of Galicia. We are going to take the 2005 matrix (base year) as a reference, and the available data for 2007 (update year) provided by the Galician Statistical Institute (IGE), as the information that is required to implement the technique. The potential uses for regional analysis of the Leontief model (in domestic flows) are the main reason for focusing on the domestic ST.

Certain modifications are required to adapt the updating method to a context like this. The order of the matrices and the dimension of the vectors change but the dynamics behind the procedure are the same as that explained. The adjustments on the intermediate consumption matrices (domestic and imported), X^d and X^m respectively, can be made at the same time. So, this means that the intermediate consumption matrix, X, includes two matrices: X^d and X^m. Equally, de final demand matrix, Y, contains Y^d and Y^m. In other words, $X \in \mathrm{M}_{2n\times n}(\Re^+)$ and $Y \in \mathrm{M}_{2n\times f}(\Re^+)$.

Another modification is made that results appropriate for applying the technique. More concretely, it is better to aggregate the imported flows into sum vectors. Thus, this gives us matrices with *n+1* rows, where the last row corresponds with the sum of the imported elements (intermediate consumption and final demand) by columns. It could be expressed as:

$$X = \begin{pmatrix} X^d \\ iX^m \end{pmatrix} \in \mathrm{M}_{(n+1)\times n}(\Re^+)$$

$$\text{and } Y = \begin{pmatrix} Y^d \\ iY^m \end{pmatrix} \in \mathrm{M}_{(n+1)\times f}(\Re^+). \qquad (31)$$

The vector of uses, e, is composed by q and m. The same applies to the intermediate demand vector, w, which is composed by w^d and w^d (*n+1* vectors). Their matrix expressions are $\begin{pmatrix} q \\ mi \end{pmatrix}$ and $\begin{pmatrix} w^d \\ w^m i \end{pmatrix}$, respectively. The intermediate consumptions come from the following addition: $u = u^d + u^m$.

It is not intended to show the entire updating process here, but it is considered appropriate to indicate the estimated change on the intermediate and final demand vectors for these two years. We display these results in the following table:

TABLE 1. INTERMEDIATE AND FINAL DEMAND VECTORS (*SURVEY* 2005 AND *NON-SURVEY* 2007)[10]

	Intermediate demand 2005	Intermediate demand 2007	Final demand 2005	Final demand 2007
R01	1.109.495	1.304.389	963.450	792.256
R02	120.054	123.734	284.636	295.142
R05	381.468	442.162	427.430	491.605
R10_12	134.201	149.978	12.091	30
R13_14	182.810	212.754	271.873	246.658
R15A	202.179	245.758	775.318	870.869
R15B	252.603	307.657	2.028.663	2.266.771
R15C	146.663	185.700	842.473	963.087
R15D	493.748	550.839	102.268	253.918
R15E	311.640	400.201	306.947	456.463
R15F_16	242.394	293.763	457.601	472.152
R17	59.611	85.063	220.616	215.159
R18	122.934	161.173	1.592.081	1.819.034
R19	6.629	8.130	48.358	51.384
R20	525.579	587.350	975.735	949.114
R21	156.233	182.225	288.386	293.219
R22	237.844	284.685	120.892	96.865
R23	531.642	634.655	1.417.070	1.565.825
R24	415.332	475.811	794.611	810.125
R25	319.879	464.540	373.912	488.842
R26	776.381	985.395	661.076	727.255
R27	556.384	830.051	1.341.425	1.805.262
R28	916.241	1.226.293	770.145	937.297
R29	385.434	497.185	324.170	435.468
R30	562	808	5.941	8.457
R31	205.810	368.642	427.433	994.709
R32	15.920	21.380	137.817	157.096
R33	18.580	21.823	25.243	25.331
R34	323.613	607.270	5.550.838	8.934.365
R35	234.721	310.083	1.111.268	1.353.277
R36	358.924	495.563	522.441	547.039
R37	31.450	41.068	2.243	10.816
R40	1.824.617	2.305.718	1.228.471	1.446.395
R41	87.177	105.784	84.945	105.747
R45	6.370.160	8.285.095	8.142.463	9.833.932
R50A	500.724	622.190	873.669	930.103
R50B	60.799	71.654	90.583	99.323
R51	1.508.293	1.875.910	2.630.973	3.311.027
R52	332.944	423.768	2.940.144	3.281.104
R55A	63.925	85.136	572.213	654.904
R55B	164.668	242.702	4.614.030	5.316.513
R60A	16.636	20.311	48.704	50.804
R60B	1.498.541	1.798.359	676.193	568.603
R61	70.494	86.961	13.369	18.124
R62	46.408	59.730	39.791	56.317
R63A	812.117	1.013.628	452.973	600.598

[10] The industrial codes correspond with the Statistical Classification of Economic Activities (NACE).

	Intermediate demand 2005	Intermediate demand 2007	Final demand 2005	Final demand 2007
R63B	34.114	45.594	227.253	287.905
R64A	118.223	139.811	26.891	23.119
R64B	944.245	1.125.570	623.194	634.304
R65	1.090.835	1.375.870	400.533	490.208
R66	266.459	331.828	438.264	508.444
R67	317.626	386.412	136.454	164.449
R70	1.740.597	2.234.910	4.061.429	4.963.386
R71	241.172	309.539	80.000	88.357
R72	146.827	196.032	57.414	108.573
R73	141.256	173.284	79.877	70.485
R74A	949.533	1.220.650	287.522	388.956
R74B	409.293	529.494	410.821	537.606
R74C	243.584	304.500	40.118	64.641
R74D	632.688	784.439	27.285	74.291
R74E	741.390	967.904	38.365	46.681
R75	-	-	3.492.371	4.086.604
R80M	89.037	112.414	524.928	589.319
R80NM	-	-	1.898.414	2.112.799
R85M	186.805	247.741	1.166.864	1.511.405
R85NM	-	-	2.278.854	2.579.374
R90M	263.990	313.365	89.956	103.422
R90NM	-	-	154.406	181.820
R91	34.515	40.891	161.330	150.318
R92M	354.694	468.945	1.061.200	1.348.256
R92NM	-	-	398.436	511.365
R93	43.116	56.953	383.339	480.020
R95	-	-	363.909	427.519

Source: Own elaboration based on the IGE data

We also emphasize, at this point, the convergence of the consecutive estimations. Although it has been necessary to make 21 iterations to achieve a complete adjustment, the approximation is more than clear in the first ones. In this way, Figure 1 shows how the distances between estimations (through R and S paths) are decreasing, as iterative phases are advancing. At the same time, there is an evident trade-off between the overestimations and the underestimations.

In the update of an IOT it is very important to guarantee convergent results. The fact of achieving convergence as well as using all the available data makes this method a useful tool. The estimations of the intermediate consumptions matrix can alternatively be expressed as an infinite multiplication of matrices, which makes the study of convergence easier. In other words, the initial intermediate consumption matrix is premultiplied and post-multiplied continuously by diagonal correction coefficients matrices. Then, as they are diagonal matrices, the task is even easier, i.e. it is possible to focus only on the convergence of the elements of the main diagonal, which are infinite multiplications of real numbers. In any case, although the correction coefficient matrices are diagonal, there is a manifest difficulty in order to express the elements of the main diagonal of the matrices that multiply by the left side. With this technique, estimations of the intermediate consumption matrix are obtained and, at the same time, estimations of the final demand matrix, linked together in the whole IO system. Therefore, this fact implies a constant change in the sum by rows of the intermediate consumption matrix, until it achieves the convergence.

4. CONCLUSIONS

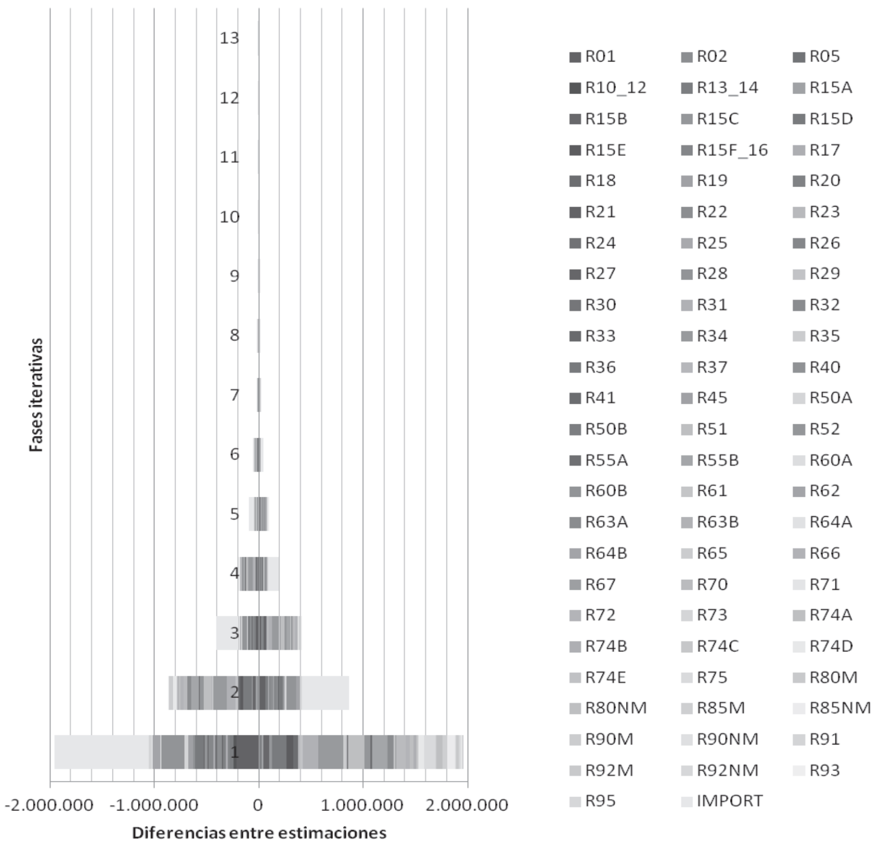
Updating matrices with partial information by distributing the differences obtained in estimations is a procedure that minimizes the time lag, uses all the available data and does not involve too complex calculations. Thus, it presents significant advantages over other adjustment methods.

For example, in order to apply the simple RAS technique more information is needed. More specifically, it is necessary to know in advance the sums by rows and columns of the matrix that it is planned to update. This constraint is an important drawback of using this method since this information is offered by the official statistical institutes with certain delay. The fact that it achieves complete updates of an IOT is a significant advancement. Consequently, regional economies that constantly need to implement and review development strategies can do it with better information.

Data about the sum by rows and columns of the intermediate consumption matrix is important but the lack of this information can be overtaken easily. There are some advantages and disadvantages using the simple RAS method but its most important drawback is surpassed with the suggested formulation. Assuming this lack of information, a technique of distribution in two ways by rows is proposed. During the process, it also achieves an important characteristic, i.e. the convergence of the results. The fact of reaching convergence as well as using all the available data makes this procedure a useful tool.

As we indicated before, this technique can correct IOT with little time lag, which implies that it can be used as a contrast instrument by official statistical institutes. It can also be used for detecting errors in the elaboration of the tables or for interpreting databases.

FIGURE 1. CONVERGENCE IN THE INTERMEDIATE DEMAND ESTIMATIONS



Source: Own elaboration

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